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Catalogue.	Epoch.	R.A. 1900·0.	P.M. applied.	N.P.D. 1900·0.	P.M. applied.
		Σ 2865. Mag. 8·5, 9·0.			
		$22^{h}\ 2^{m}$ s	s	20° 16′ ″	″
Lalande F. ...	1789·7	10·52	11·90	31·3	29·1
Pos. Med. ...	1823·6	10·62	·57	29·6	28·1
Oeltz. Arg. (N.)...	41·6	10·50	·23	31·8	30·6
Brussels	67·3, 67·8	11·17	·58	28·7	28·1
Christiania ...	75·5	11·12	·43	27·6	27·1
Greenwich ...	99·2	11·52	·53	28·2	28·2

P.M. of A +0″·064 −0″·020

The relative motion of B to A from micrometer measures :

−0″·072 +0″·054

P.M. of B −0″·008 +0″·034

Note on the Determinations of Positions and Magnitude of Stars in the Greenwich Astrographic Catalogue.

(*Communicated by the Astronomer-Royal.*)

The Introduction to vol. i. of the Greenwich Section of the *Astrographic Catalogue*, which is now in the press and will shortly be published, contains short discussions of the personality of the measurers, the probable error of the measures and of resulting right ascensions and declinations, and of the relation between magnitude of stars and the diameters of their photographic images. It may be of interest to lay before the Society a brief summary of these investigations.

I. *Personality of Measurers.*

The duplicate measurement of the plates in reversed positions, with a view to an increase of accuracy, was undertaken as a result of the meeting of the Astrographic Committee at Paris in 1896 June. Zones 64°, 65°, 66°, 67°, had by this time been measured at Greenwich in the direct positions of the plates. They were accordingly re-measured, the plates and the glass diaphragm being reversed right for left. The direct and reversed measures were compared, the investigation involving many thousands of measures of about 300 plates by eight different measurers. In the direct measures the same measurer measured both the 6^{m} and 3^{m} images, but in the reversed measures there were separate measurers for the two images.

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The means of the measures of the 6^{m} and 3^{m} images having been already formed, the differences of these means direct and reversed were tabulated for each combination of measurers, and for convenience each book of measures was dealt with separately. As an example the following table is given, showing the result of the collection of these differences for the second book of zone 66°, extending from R.A. 6^{h} 0^{m} to 11^{h} 24^{m}. It should be stated that the "direct" measures of this book were made in 1896 March and the "reversed" measures in 1897 April.

Differences of Measures Direct–Reversed as made by different Observers for Zone 66° *R.A.* 6^{h} 0^{m}–11^{h} 24^{m}.

Unit = ·0001 of a réseau interval = ″·03.

Measurers.				Centre +66° (South halves).				Centre +67° (North halves).			
Direct.		Reversed.		*x*		*y*		*x*		*y*	
6^{m}	3^{m}	6^{m}	3^{m}	No. of Stars.	Sum of Diffs.	No. of Stars.	Sum of Diffs.	No. of Stars.	Sum of Diffs.	No. of Stars.	Sum of Diffs.
J.,	J.	J.,	P. M.	36	+536	39	+288	59	+1214	59	+273
J.,	J.	W. S.,	P. M.	6	+ 35	6	+ 76	8	+ 96	8	+ 33
J.,	J.	C. D.,	P. M.	15	+142	15	+792	29	+210	29	− 49
W. S.,	W. S.	J.,	P. M.	32	+249	32	−219	57	−286	57	− 1
W. S.,	W. S.	J.,	E. S.	37	+192	37	−292	29	−145	29	− 35
W. S.,	W. S.	W. S.,	P. M.	67	+316	65	−672	67	−824	68	− 76
W. S.,	W. S.	W. S.,	E. S.	102	+692	102	−1087	104	−1503	107	+669
W. S.,	W. S.	P. M.,	E. S.	16	+ 62	16	− 96	17	−199	18	+ 72
W. S.,	W. S.	C. D.,	J.	18	+132	19	−221	19	+ 65	20	− 21
P. M.,	P. M.	J.,	W. S.	29	− 90	27	+ 21	38	−202	36	+ 27
P. M.,	P. M.	J.,	P. M.	18	−198	19	+ 32	55	−330	55	+261
P. M.,	P. M.	W. S.,	E. S.	50	−293	49	−525	47	−561	47	+290
P. M.,	P. M.	C. D.,	P. M.	23	−286	23	− 71	19	−264	19	+ 27
E. S.,	E. S.	W. S.,	E. S.	53	+ 53	52	−526	61	−680	62	−701
E. S.,	E. S.	C. D.,	P. M.	3	− 8	3	− 21	4	+ 47	4	− 38
E. S.,	E. S.	J.,	W. S.	7	+ 41	7	−103	10	− 57	10	− 86
E. S.,	E. S.	J.,	P. M.	31	+206	31	− 57	30	+ 5	29	−378
E. S.,	E. S.	J.,	E. S.	20	+ 89	20	− 69	25	+ 48	25	− 8
Total No. of Stars ...				563		562		678		682	

Denoting by the subscripts 1 and 2 measures made in the direct and reversed positions respectively, the above table gives equations of condition of the form

$$36\left(J._{1}-\frac{J._{2}+P.M._{2}}{2}\right)=+536$$

Normal equations were formed and solved in the usual way. The zero being arbitrary, it was assumed so that $J._{1}+J._{2}+W.S._{1}$

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+W.S.$_2$=0, as J. and W.S. had measured for a long period. The solution of these equations gave the following values for the personalities of the observers, in which a distinction is made according to the microscope of the duplex micrometer used :

	J.$_1$	J.$_2$	W.S.$_1$	W.S.$_2$	P.M.$_1$	P.M.$_2$	E.S.$_1$	E.S.$_2$
x Left-hand microscope	+ 9	− 7	+ 3	−4	−10	+1	− 2	0
x Right-hand microscope	+16	−11	−10	+6	− 9	+2	− 5	+3
y Left-hand microscope	+ 6	− 5	− 8	+5	− 4	−1	− 6	+4
y Right-hand microscope	+ 1	− 1	− 5	+4	+ 3	−7	−11	−7

Nineteen books of measures were treated in this manner, and the results, which are given in the Introduction to the Greenwich Astrographic Catalogue, show that these personalities were fairly constant. The mean results (with changed signs) are given in the following table of corrections. Different measures show clearly that the mean of the corrections, direct and reversed, is nearly zero for each measurer ; that is to say, the mean of a direct and reversed measure by the same measurer is almost free from personality.

Corrections for Personality.

Unit = $^{\text{int.}}$·0001 = 0″·03.

Measurer.	Measures of x Coordinate. Right-hand Microscope. Direct.	Reversed.	Left-hand Microscope. Direct.	Reversed.	Measures of y Coordinate. Both Microscopes. Direct.	Reversed.
Miss Everett (A.E.)	+ 3	...	+1	...	+5	...
Mr. Davidson (C.D.)	+ 5	+2	+3	+ 1	+6	−3
Mr. Johns (J.) ...	−11	+8	−9	+ 7	+1	+1
Mr. Stevens (W.S.)	+ 6	−5	−2	+ 3	+6	−5
Mr. Melotte (P.M.)	+ 3	−2	+3	− 1	−4 or +2*	+4 or −2*
Mr. Skells (E.S.)...	+ 3	−7	+2	+ 2	0	+4
Mr. Evans (E.) ...	+ 1	−4	...	+ 9	0	0
Mr. Stiles (St.) ...	+13	−9	0	−12	−4	−5

As a result of this investigation it was arranged that the same measurer should measure the 6$^{\text{m}}$ images in the direct and reversed positions of the plate, and another measurer the 3$^{\text{m}}$ images also in both positions.

II. *Probable Error of the Measures.*

In 1896 July two investigations were made of the probable error of the measures, including the errors inherent in the photographic images of a star and the réseau lines to which it is referred.

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* The values +2 and −2 were found for some measures made (direct and reversed) at a later date in 1897 June–August.

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(i.) Measures of the same plate were made by five measurers, each measurer measuring both the 6^m and 3^m images on the plate in direct and reversed positions.

(ii.) Five plates of the same field taken at different times and under different conditions were measured by one measurer in direct and reversed positions, giving ten measures.

(i.) The values for the discordance of the mean of the measures of a 6^m and 3^m image by one observer in one position (direct or reversed), corrected for personality, from the mean of ten such measures of 113 stars was found to be

Measurer.		*x* Coordinate. Direct.	*x* Coordinate. Reversed.	*y* Coordinate. Direct.	*y* Coordinate. Reversed.
		″	″	″	″
Mr. Davidson	...	±·120	±·135	±·150	±·195
Mr. Johns	...	±·219	±·186	±·219	±·174
Mr. Stevens	...	±·177	±·174	±·171	±·159
Mr. Melotte	...	±·231	±·222	±·219	±·219
Mr. Skells	...	±·213	±·186	±·186	±·183

The means of the above discordances are ±″·186 and ±″·187 in the two coordinates respectively, leading in each case (after multiplication by $\sqrt{\frac{10}{9}} \times \cdot 845$) to a probable error of ±″·166. Thus the probable error of a complete measure (two images, direct and reversed) is ±″·117.

It is to be noted that a complete measure involves eight readings or estimations of the positions of the two réseau lines relatively to the star images, these being practically quite independent, as owing to irregular curvature of the plate the "run" for a réseau interval varies from point to point.

If, instead of applying a correction for personality, the means of the measures, direct and reversed, are taken for each observer, the discordances are found to be

Measurer.				*x* Coordinate.	*y* Coordinate.
				″	″
Mr. Davidson	...	...	...	±·108	±·131
Mr. Johns	...	...	...	±·153	±·159
Mr. Stevens	...	...	...	±·114	±·126
Mr. Melotte	...	...	...	±·132	±·141
Mr Skells	...	...	...	±·144	±·132

The probable error of a complete measure (multiplying the mean discordance by $\sqrt{\frac{5}{4}} \times \cdot 845$) is found to be ±″·127. This agrees satisfactorily with the result given above, showing that personality of the measurer is virtually eliminated in the mean of measures direct and reversed.

(ii.) Five plates of the same field were completely measured by Mr. Stevens, direct and reversed. In each case the means of the measures of the 6^m and 3^m images were taken. The plate

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constants were obtained exactly as is done in practice by comparing the measured coordinates with the standard coordinates of the reference stars of which there were twenty-two on the plate. Corrections of the form $ax+by+c$, $dx+ey+f$ were applied to the measured coordinates, thus giving ten comparable values of the coordinates of 121 stars. The means of the ten measures, direct or reversed, were taken for each star, and the discordances from the mean formed. The mean discordances are given in the following table:

Plate No.	*x* Coordinate.		*y* Coordinate.	
	Direct.	Reversed.	Direct.	Reversed.
	″	″	″	″
419	±·42	±·35	±·34	±·31
3139	±·25	±·35	±·20	±·29
3150	±·34	± 26	±·19	±·24
3151	±·28	±·25	±·21	±·36
3089	±·27	± 29	±·26	±·24

The mean of these discordances is ±″·285, giving ±″·254 for the probable error of a single determination of position (6^m and 3^m image) and ±″·180 of a complete determination, direct and reversed.

If the discordances of the means of the direct and reversed measures are taken for the five plates, the results are:

Plate No.	*x* Coordinate.	*y* Coordinate.
	″	″
419	±·34	±·30
3139	±·18	±·17
3150	±·27	±·20
3151	±·20	±·23
3089	±·25	±·21

and the probable error of a complete measure deduced in this way is ±″·222. This increase in the probable error may be taken as due to the systematic error which is common to the plate, both for direct and reversed measures.

It should be noted that in this probable error are included (i.) the errors of the measures, (ii.) the errors inherent in the photographic images of stars and réseau lines, (iii.) the errors introduced through errors in the plate constants arising from these causes, but not the part arising from the assumed right ascensions and declinations of the reference stars.

A further discussion of probable errors of places of the stars deduced from the photographs has been made by comparing the separate results from pairs of plates for a limited number of stars. The right ascensions and declinations of 240 stars between 70° N.Dec. and 72° N.Dec. have been deduced, with the provisional plate constants given in the Introduction to the Catalogue, one

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star being taken in each quarter of the overlapping plates in this zone. A comparison of the results from the two plates on which each star was shown was made, the mean differences being found to be $\pm''\cdot43$ in R.A. and $\pm''\cdot46$ in Dec. From this it results that the probable errors (in arc of a great circle) of the right ascension and declination of a star deduced from the measures on one plate are $\pm''\cdot26$ in R.A. and $\pm''\cdot28$ in Dec., and that the probable errors of a catalogue place from the mean of two plates may be taken as $\pm''\cdot19$ in both R.A. and declination. These results were obtained with provisional places of the reference stars, the accidental errors of which affect the plate constants to some extent. In order to obtain more accurate places of the reference stars for determination of plate constants, these stars are being re-observed with the transit-circle at Greenwich; and when these observations are completed in the course of a couple of years the accuracy of the plate constants will be materially increased. From a discussion of the observations of the reference stars from N.P.D. 0° to 5°, now completed, it appears that the probable error of a star's place in the new Greenwich Meridian Catalogue (five observations in each element) is $\pm 0''\cdot23$ in R.A. or N.P.D.

III. *Determination of Photographic Magnitudes.*

In the publication of the Greenwich Measures the diameter of the 6^{m} image is given as a measure of the photographic magnitude. In each case the diameter is estimated by two measurers in divisions and tenths of a division of the scale and the sum taken. Allowance is made for images which are sensibly elongated by taking the mean of the diameters in the horizontal and vertical directions. Allowance is also made by the measurers for the want of density in the images and also for the shading off at the edge, and the diameter set down is that of an equivalent black image, *i.e.* one which would contain the same amount of silver deposit.

In order to obtain direct information as to the magnitude of the faint stars shown on the plates, comparison has been made with the determinations on a photometric scale given by Professor E. C. Pickering (*Harvard Annals*, vol. xxxvii. pp. 4–11) of the faint stars near certain variables which lie in the Greenwich section. The photometric magnitudes of faint stars near the pole (*ibid.*, vol. xviii. p. 134) have also been compared with the diameters of the images on a catalogue plate (exp. 6^{m}, 3^{m}, and 20^{s}) and on four chart plates (exp. 40^{m}). Some photographs of *Nova Persei* taken under the same conditions as the astrographic plates also admit of comparison with the magnitudes given by Father Hagen (Georgetown College Observatory, *Supplementary Notes to the Atlas Stellarum Variabilium*).

The following table, giving the stars near *R Draconis* R.A. $16^{h}\ 32^{m}\ 23^{s}$, Decl. $+66°\ 57'\cdot8$ (1900·0), shown on plates with

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centres at R.A. 16^h 30^m, Decl. + 67°, may serve as a specimen of the comparisons made, which deal with 19 chart plates and 18 catalogue plates of seven different fields.

Diameters of Images of Stars near R Draconis.

Unit = 0″·15.

Reference Letter.	Photom. Mag.	Plate 2652. Exp. 20^s.	Plate 2652. Exp. 6^m.	Plate 3482. Exp. 40^m.	Zone and No. in Greenwich Measures.
a	6·88	29	74	121	67°, 5303
c	7·44	28	70	117	67, 5299
f	8·28	14	43	82	66, 5250
g	8·56	20	51	84	67, 5395
h	9·06	12	42	80	67, 5283
k	9·40	7	32	62	66, 5248
l	9·80	6	20	52	67, 5317
m	10·16	5	28	54	67, 5370
n	10·56	5	24	46	66, 5223
o	10·97	not shown	15	34	66, 5222
p	11·59	,,	9	26	66, 5251
q	12·01	,,	8	20	66, 5234
r	12·52	,,	6	20	66, 5254
s	12·85	,,	not shown	18	...
t	13·33	,,	,,	13	...
u	13·84	,,	,,	15	...
w	14·43	,,	,,	13	...
x	15·02	,,	,,	10	...

The faintest stars shown with an exposure of 6^m on all the plates where a comparison with photometric magnitudes could be made are shown in the following table :—

Plate.	Mag. m	Plate.	Mag. m	Plate.	Mag. m
4061	12·61	4027	12·92	6160	11·6
1678	11·08	2652	12·52	6168	12·7
4767	11·87	2660	12·01	6178	11·9
3409	12·02	2776	11·96	6200	11·8
2469	12·02	2771	11·96	6202	12·8
2708	12·92	5013	13·2	6204	12·7

The mean for the above plates is 12^m·25, and this may be taken as an approximate value of the average limit of magnitude reached by the catalogue plates at Greenwich with an exposure of 6 minutes.

For the conversion of diameters into magnitudes the em-

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pirical law connecting the measured diameter d and the magnitude m determined by the Astronomer Royal (*Monthly Notices*, vol. lii. pp. 125–146), viz.:—$m=C-n\sqrt{d}$, has been adopted and appears to give a satisfactory accordance with the photometric determinations. The following determinations of the values of the constants C and n are derived by direct comparison of the bright and faint stars on each of 19 chart plates (exp. 40^{m}) :—

Plate.	Field.	Range of magnitudes and No. of stars.				Deduced formula $m=C-n\sqrt{d}$.	Diameter given by formula.				
		m		m			$9^{m}{\cdot}0$	$11^{m}{\cdot}0$	$13^{m}{\cdot}0$	$14^{m}{\cdot}0$	$15^{m}{\cdot}0$
1517	S. Cassiop.	8·76	1	13·93	3	$15{\cdot}7-0{\cdot}68\sqrt{d}$	97	48	16	6	1
5800	,,	8·76	1	13·93	3	$15{\cdot}7-0{\cdot}68\sqrt{d}$	97	48	16	6	1
3719	,,	9·92	2	14·12	2	$16{\cdot}0-0{\cdot}75\sqrt{d}$	87	44	16	7	2
3878	R. Urs. Maj.	8·49	2	14·01	3	$16{\cdot}7-0{\cdot}74\sqrt{d}$	109	59	25	13	5
3436	,,	7·94	2	14·01	3	$15{\cdot}8-0{\cdot}85\sqrt{d}$	64	32	11	5	1
4019	R. Urs. Min.	8·76	4	12·55	3	$16{\cdot}6-0{\cdot}90\sqrt{d}$	71	40	16	8	3
3515	,,	8·94	1	12·55	3	$16{\cdot}1-0{\cdot}87\sqrt{d}$	67	35	13	6	2
3469	R. Draconis	8·91	3	14·43	3	$17{\cdot}2-1{\cdot}00\sqrt{d}$	67	38	18	10	5
3482	,,	7·16	2	14·43	3	$17{\cdot}6-1{\cdot}00\sqrt{d}$	74	44	21	13	7
4587	T Cephei	8·46	2	12·55	2	$16{\cdot}4-0{\cdot}87\sqrt{d}$	74	40	16	8	3
3223	,,	8·12	1	12·55	2	$16{\cdot}3-0{\cdot}81\sqrt{d}$	82	43	17	8	3
6058	Pole	10·00	4	14·40	4	$16{\cdot}6-0{\cdot}95\sqrt{d}$	64	35	15	7	3
6050	,,	10·00	4	14·40	4	$16{\cdot}2-0{\cdot}78\sqrt{d}$	85	45	17	8	2
5955	,,	10·00	4	14·40	4	$16{\cdot}1-0{\cdot}95\sqrt{d}$	56	29	11	5	1
5964	,,	10·00	4	14·40	4	$16{\cdot}4-0{\cdot}98\sqrt{d}$	57	31	12	6	2
6200	Nova Persei	7·70	5	12·50	9	$14{\cdot}6-0{\cdot}77\sqrt{d}$	53	22	4	1	...
6202	,,	7·70	5	12·50	9	$16{\cdot}0-0{\cdot}79\sqrt{d}$	79	40	14	6	2
6204	,,	7·70	5	12·50	9	$15{\cdot}2-0{\cdot}84\sqrt{d}$	54	25	7	2	1
6161	,,	7·70	5	12·50	9	$14{\cdot}7-0{\cdot}78\sqrt{d}$	53	22	5	1	...

The mean formula derived from the chart plates (exposure 40^{m}) is $m=16{\cdot}1-0{\cdot}84\sqrt{d}$.

Formulæ were derived in a similar way for the exposures of 6^{m}, 3^{m}, and 20^{s} of the catalogue plates, the mean results being as follows :—

$$\left.\begin{array}{lll} \text{m} & & \\ m=16{\cdot}1-0{\cdot}84\sqrt{d} & \text{for exposure} & 40^{m}\ (19\text{ plates}) \\ m=13{\cdot}7-0{\cdot}77\sqrt{d} & \text{,,}\qquad\text{,,} & 6^{m}\ (18\text{ plates}) \\ m=12{\cdot}9-0{\cdot}76\sqrt{d} & \text{,,}\qquad\text{,,} & 3^{m}\ (6\text{ plates}) \\ m=11{\cdot}5-0{\cdot}79\sqrt{d} & \text{,,}\qquad\text{,,} & 20^{s}\ (9\text{ plates}) \end{array}\right\}$$

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Putting $d=4$ as practically the diameter of a star which is shown quite clearly (though there are a number of stars measured with diameters 3 or 2), we find

m
$m=14{\cdot}4$ for the average limiting magnitude with exposure $40^{\rm m}$
$m=12{\cdot}3$ „ „ „ „ $6^{\rm m}$
$m=11{\cdot}4$ „ „ „ „ $3^{\rm m}$
$m=\ 9{\cdot}9$ „ „ „ „ $20^{\rm s}$

The following table gives the observed difference of magnitude in passing from one exposure to another compared with the values of $2{\cdot}5 \log t_2/t_1$:

	Obs. Diff.	$2{\cdot}5 \log \frac{t_2}{t_1}$.
	m	m
$20^{\rm s}$ to $3^{\rm m}$	1·5	2·39
$3^{\rm m}$ to $6^{\rm m}$	0·9	0·75
$6^{\rm m}$ to $40^{\rm m}$	2·1	2·06
$20^{\rm s}$ to $40^{\rm m}$	4·5	5·20

It appears from this that in passing from $3^{\rm m}$ to $6^{\rm m}$, and again from $6^{\rm m}$ to $40^{\rm m}$, the law

$$\text{Exposure} \times \text{brightness} = \text{constant}$$

holds almost exactly, while the exposure of $20^{\rm s}$ gives fainter stars than would be expected in accordance with the law. As a possible explanation of this it is to be noted that the $20^{\rm s}$ exposure was given after the exposures of $6^{\rm m}$ and $3^{\rm m}$, and that this preliminary exposure to diffused light may have increased the sensitiveness of the plate for the $20^{\rm s}$ exposure.

In a paper on the statistics of stars in a zone of 5° from +65° to +70° Dec. (*Monthly Notices*, 1903 January), the logarithms of the number of stars per square degree with different exposures are given as follows :—

I. *Shown on both plates.*

$20^{\rm s}$	$3^{\rm m}$	$6^{\rm m}$	$40^{\rm m}$
0·966	1·598	1·724	2·437

II. *Total number shown.*

$20^{\rm s}$	$3^{\rm m}$	$6^{\rm m}$	$40^{\rm m}$
1·186	1·818	1·830*	2·496*

Between the $20^{\rm s}$ and $3^{\rm m}$ exposure the log increase of number

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* As stated in the above paper the number 1·830 is too small, as stars which show a $6^{\rm m}$ image on one plate only are not counted. The number 2·496 is probably too small, though not to the same extent.

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of stars per square degree is 0·632 or (the difference of magnitude being 1·5) at the rate of 0·42 per magnitude.

Between 20^s and 6^m the log increase of number of stars per square degree is 0·757, or (the difference of magnitude being 2·4) at the rate of 0·32 per magnitude.

Taking the mean of these results we find that between the magnitudes 10^m·0 and 12^m·0 the logarithmic increase per magnitude in the total number of stars is 0·37, and therefore the ratio of increase per magnitude is 2·34.

Between 3^m and 40^m there is a logarithmic increase of 0·839, corresponding to a difference of magnitude of 3^m·0 or 0·28 per magnitude, and between 6^m and 40^m of ·713, corresponding to a difference of magnitude of 2^m·1 or of 0·34 per magnitude.

Taking the mean of these results we have between 12^m·0 and 14^m·4 a logarithmic increase of number of stars per magnitude =0·31, corresponding to a ratio of increase of 2·04 per magnitude.

It is satisfactory to find that these results are in good accordance with those recently given by Professor Pickering in his valuable memoir on the "Distribution of Stars" (*Harvard Annals*, vol. xlviii. No. 5, p. 178). Taking the values given in his Table XXI., the logarithmic increase per magnitude in the total number of stars would be 0·39 (as compared with 0·37 found above) between 10 and 12 mag., and 0·31 (as compared with 0·31) between 12^m·0 and 14^m·4, the corresponding ratios of increase per magnitude in the total number of stars being 2·45 and 2·04 found by Professor Pickering, as compared with 2·34 and 2·04 found above.

Royal Observatory, Greenwich:
1904 *March* 8.

On the Chromatic Correction of Object-glasses. Second Paper.

By A. E. Conrady.

In my first paper [*] I limited myself to the consideration of a luminous point in the optical axis. Precisely the same reasoning may, however, be applied to any point in the field of an object glass; but as in the case of oblique pencils there is no such complete symmetry as in axial pencils, a greater number of rays have to be followed through the system of lenses and their paths in the successive media determined in order to compare the value of $\Sigma dn D$ for the several rays, and thus to determine the state of the chromatic correction.

It will generally be sufficient to compute four rays through the system: three of these lie in the plane containing the optical axis and the luminous point; these should be computed

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[*] *Monthly Notices*, January 1904.